Acta Technica *56* (2011), T175–T182

Brillouin back scattering in vertically magnetized plasmas

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Abstract. Brillouin scattering occurs in the interaction of picosecond laser pulse with a plasma which involves the coupling of large amplitude light wave into a scattered light wave plus an ion acoustic wave. Previous researches have shown that this scattering leades to instability with specific growth rate in a plasma. In this paper, we investigated Brillouin Back scattering instability growth rate in a magnetized plasma. As a research, plasma is imbedded in an uniform external vertically magnetic field. It is shown that this magnetic field alters the growth rate significantly.

Key Words. Brillouin scattering, Brillouin instability, ion acoustic wave, growth rate.

Introduction

Stimulated Brillouin scattering (SBS) plays an important role in laser–plasma interaction as it produces a backscattered light, and therefore this process is one of the real threat to the inertial confnement fusion research. After the invention of the ruby laser in 1960, Chiao et al. [7] was the first to observe SBS.

The control of the stimulated Brillouin scattering (SBS) instability remains one of the key issues for the success of laser fusion. In the context of laser-plasma interaction, incident laser wave couples to an ion-acoustic wave (IAW) to give rise to a scattered transverse wave, leading to a net energy loss in the case of backscattering [9].

Stimulated Brillouin scattering in a plasma is the decay of an incident (pump) light wave into a frequency downshifted (Stokes) light wave and an ion-acoustic (sound wave). It is important in direct and indirect inertial confinement fusion (ICF) experiments because it scatters the laser beams away

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from the target, thereby reducing the energy available to drive the compressive heating of the nuclear fuel [1], [4].

Parametric Instabilities

Large amplitude waves encountered in plasma-based sources of coherent radiation are susceptible to parametric instabilities. In a parametric process a large amplitude pump wave couples to a low-frequency mode of the plasma to produce a sideband wave. The pump and the sideband wave exert a nonlinear force on the plasma particles to drive the low-frequency mode. In this process the amplitudes of the low-frequency mode and the sideband, called the daughter waves, grow with time once the pump power exceeds a threshold value, set by the linear damping or convective losses of the daughter waves. The parametric instability saturates via pump wave depletion or a downward cascade of energy from the sideband wave. In a plasma-based source of coherent radiation, parametric instabilities should lead to frequency broadening of the beam driven mode. Hence an in depth study of parametric instabilities in plasma-filled devices is of considerable importance. Parametric instabilities have been studied extensively in laser-produced plasmas where stimulated Raman and Brillouin scattering (SRS) and (SBS) are two dominant parametric processes.

In a SRS process an intense laser beam drives a Langmuir wave and an electromagnetic sideband wave. The instability occurs at densities below quarter critical. However, the density of the plasma should not be too small otherwise the Langmuir wave is strongly Landau-damped by the electrons. The growth rate peaks when the sideband wave propagates opposite to the pump wave. In the SBS process, the laser excites an ion acoustic wave and an electromagnetic sideband wave. The instability occurs over a wide range of electron density, up to the critical layer. Nevertheless, it requires nonisothermal plasma where the electron temperature is much larger than the ion temperature, otherwise the ion Landau damping will suppress the Brillouin instability.

Stenflo has developed an elegant theory of parametric instabilities including the effect of a magnetic field [10].

The Brillouin instability can be most simply characterized as the resonant decay of an incident photon with frequency ω_0 and wave number k_0 into a scattered photon with frequency ω_{s} and wave number k_{s} plus an ion acoustic phonon. The frequency and wave number matching conditions then are $(\omega_0 = \omega_{\mathrm{s}} + \omega)$ and $(\boldsymbol{k}_0 = \boldsymbol{k}_{\mathrm{s}} + \boldsymbol{k})$. Where now ω and $\boldsymbol{k}$ are the frequency and wave number of the ion acoustic wave. Since the frequency of an ion acoustic wave is much less than ω_0, it is clear that this instability can occur throughout the underdense plasma. Furthermore, nearly all the energy can be transferred to the scattered light wave.

Instability Analysis

To obtain the coupled equation describing the Brillouin instability, we consider the response of an initially uniform plasma driven by a large amplitude light wave. For clarity we consider an ordinary light wave propagating through a plasma with a uniform density and temperature. Plasma is imbedded in a constant magnetic field $\boldsymbol{B}_0$ in $\hat{y}$ direction. Electric field of electromagnetic wave has the form of $\boldsymbol{E} = \hat{e}_x E(z,t)\cos(k_0 z - \omega_0 t)$ and its magnetic field $\boldsymbol{B}_0$ is in the $\hat{y}$ direction. We can ignore the magnetic field of the light wave in comparison with strong external magnetic field. Thus, the wave equation and the force equation of electrons are respectively

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\boldsymbol{E} = \frac{4\pi}{c}\frac{\partial \boldsymbol{J}}{\partial t}, \tag{1}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{e}{m}\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}_0\right). \tag{2}$$

Equation (2) has two components,

$$\frac{\partial v_x}{\partial t} = -\frac{e}{m}E_x + \omega_\mathrm{c} v_z, \tag{3}$$

$$\frac{\partial v_z}{\partial t} = -\omega_\mathrm{c} v_x, \tag{4}$$

where $\omega_\mathrm{c} = eB_0/m$, is the electron cyclotron frequency. We solve this equations to obtain (5) and (6) as below:

$$v_\perp = v_x = \frac{ca\omega^2}{(\omega^2 - \omega_\mathrm{c}^2)}\sin(kz - \omega t), \tag{5}$$

$$v_z = -\frac{ca\omega\omega_\mathrm{c}}{(\omega^2 - \omega_\mathrm{c}^2)}\cos(kz - \omega t). \tag{6}$$

Thus we can write the current density of electrons as

$$J_x = -nev_\perp = -enca\left[\frac{\omega^2}{(\omega^2 - \omega_\mathrm{c}^2)}\right]\sin(kz - \omega t). \tag{7}$$

Hence the equation (1) becomes

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\boldsymbol{A} = -\frac{4\pi n_\mathrm{e} e^2}{m_\mathrm{e}}\left[\frac{\omega^2}{(\omega^2 - \omega_\mathrm{c}^2)}\right]\boldsymbol{A}, \tag{8}$$

where $\boldsymbol{A}$ is the vector potential of the light wave and $\boldsymbol{a} = e\boldsymbol{A}/mc^2$ is the normalized potential vector. To obtain the coupled equations describing the Brillouin instability, we consider the response of an initially uniform plasma driven by a large amplitude light wave. We can write equation for the generation of scattered light wave with vector potential $\tilde{A}$ by the coupling of a large amplitude light wave with vector potential A_{l} with an electron fluctuation $\tilde{n}_{\mathrm{e}}$. Hence current density can be taken as $\tilde{\boldsymbol{J}} = -ne\tilde{\boldsymbol{v}}_{\perp} - \tilde{n}e\boldsymbol{v}_{\perp 0}$. Then equation (1) has the form of

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_{\mathrm{pe}}^2\left[\frac{\omega^2}{(\omega^2-\omega_{\mathrm{c}}^2)}\right]\right)\tilde{\boldsymbol{A}} = -\frac{4\pi\tilde{n}_{\mathrm{e}}e^2}{m_{\mathrm{e}}}\left[\frac{\omega_0^2}{\omega_0^2-\omega_{\mathrm{c}}^2}\right]\boldsymbol{A}_{\mathrm{l}}, \tag{9}$$

where ω_{pe} is the electron plasma frequency and ω is the frequency of scattered light wave. Only the fluctuation in electron density appears in Eq. (9), since the ion response to the light frequency field of the light wave is less than the electron response by Zm/M, where Z is the charge state, m the electron mass, and M the ion mass. For the Brillouin instability, the density fluctuation $\tilde{n}_{\mathrm{e}}$ is the low frequency fluctuation associated with an ion acoustic wave. We describe the electrons as a warm fluid and separate the fluid velocity v_{e} into longitudinal (u_{l}) and transverse components (eA/mc). Then by the taken of continuity and force equations

$$\frac{\partial\tilde{n}_{\mathrm{e}}}{\partial t} + n_0\nabla\cdot\vec{\boldsymbol{v_e}} = 0,$$
$$\frac{\partial\tilde{v}_{\mathrm{e}}}{\partial t} + \boldsymbol{v}_{\mathrm{e}}.\nabla\tilde{v}_{\mathrm{e}} = -\frac{e}{m}(\boldsymbol{E} + \frac{\boldsymbol{v}_{\mathrm{e}}\times\boldsymbol{B}}{c}) - \frac{\nabla\tilde{p}}{n_{\mathrm{e}}m_{\mathrm{e}}},$$

we obtain

$$\frac{\partial\boldsymbol{u}_{\mathrm{l}}}{\partial t} = \frac{e}{m}\nabla\varphi - \frac{1}{2}\nabla\left(\boldsymbol{u}_{\mathrm{l}} + \frac{e\boldsymbol{A}}{mc}\right)^2 - \frac{\nabla p_{\mathrm{e}}}{n_{\mathrm{e}}m_{\mathrm{e}}}, \tag{10}$$

where φ is the electrostatic potential, p_{e} the electron pressure, and n_{e} the electron density. Since we are now considering a low frequency fluctuation, we neglect the electron inertia ($\partial u_{\mathrm{l}}/\partial t \to 0$) and use the isothermal equation of state ($p_{\mathrm{e}} = n_{\mathrm{e}}\theta_{\mathrm{e}}$, where $\theta_{\mathrm{e}} = T_{\mathrm{e}}$ is the electron temperature). We then linearize Eq. (10) by letting $n_{\mathrm{e}} = n_0 + \tilde{n}_{\mathrm{e}}$, $\boldsymbol{A} = \boldsymbol{A}_{\mathrm{l}} + \tilde{\boldsymbol{A}}$ and $\varphi = \tilde{\varphi}$, which gives

$$\frac{e}{m}\nabla\tilde{\varphi} = \frac{e^2}{m^2c^2}\nabla\left(\boldsymbol{A}_{\mathrm{l}}\cdot\tilde{\boldsymbol{A}}\right) + \frac{v_{\mathrm{e}}^2}{n_0}\nabla\tilde{n}_{\mathrm{e}}. \tag{11}$$

The electrical potential transmits the ponderomotive force to the ions. To treat the ion response, we describe the ions as a charged fluid with density n_{i} and velocity u_{i}. The continuity and force equations of ions are

$$\frac{\partial n_{\mathrm{i}}}{\partial t} + \nabla\cdot(n_{\mathrm{i}}\boldsymbol{u}_{\mathrm{i}}) = 0,$$

$$\frac{\partial \boldsymbol{u}_{\rm i}}{\partial t} + \boldsymbol{u}_{\rm i} \cdot \nabla \boldsymbol{u}_{\rm i} = -\frac{Ze}{M} \nabla \varphi,$$

where we have neglect the ion pressure for simplicity. We next linearize these equations by taking $n_{\rm i} = n_{0\rm i} + \tilde{n}_{\rm i}$, $u_{\rm i} = \tilde{u}_{\rm i}$ and $\varphi = \tilde{\varphi}$. Then

$$\frac{\partial \tilde{n}_{\rm i}}{\partial t} + n_{0\rm i} \nabla \cdot \vec{\boldsymbol{u}}_{\rm i} = 0, \tag{12}$$

$$\frac{\partial \bar{\tilde{u}}_{\rm i}}{\partial t} = -\frac{Ze}{M} \nabla \tilde{\varphi}. \tag{13}$$

Taking a time derivative of Eq. (13), a divergence of Eq. (12) and combining then gives

$$\frac{\partial^2 n_{\rm i}}{\partial t^2} - \frac{n_{0\rm i} Ze}{M} \nabla^2 \tilde{\varphi} = 0.$$

If we substitute for $\tilde{\varphi}$ using Eq. (11), we finally obtain an equation for the low frequency density fluctuation

$$\frac{\partial^2 \tilde{n}_{\rm e}}{\partial t^2} - c_{\rm s}^2 \nabla^2 \tilde{n}_{\rm e} = \frac{Z n_0 e^2}{m M c^2} \nabla^2 \left(\boldsymbol{A}_{\rm l} \cdot \tilde{\boldsymbol{A}} \right). \tag{14}$$

Hence $A_{\rm l} = A_{\rm l} \cos(k_0 z - \omega_0 t)$ is the ion acoustic velocity. Eq. (9) describing the excitation of an ion acoustic wave by the interaction between the incident and scattered light waves.

Dispersion Relation

By taking the $A_{\rm l} = A_{\rm l} \cos(k_0 z - \omega_0 t)$ for incident light wave and using the Fourier-analyze Eq. (9) and (14), then we can obtain the dispersion relation from the coupled equations for $\tilde{A}$ and $\tilde{n}_{\rm e}$.

$$D(k,\omega)\tilde{A}(k,\omega) = \frac{4\pi e^2}{m_e} \left[\frac{\omega_0^2}{\omega_0^2 - \omega_{\rm c}^2} \right] \frac{A_{\rm l}}{2} \times \times \left[\tilde{n}_{\rm e}(k - k_0, \omega - \omega_0) + \tilde{n}_{\rm e}(k + k_0, \omega + \omega_0) \right] \tag{15}$$

$$(\omega^2 - c_{\rm s}^2 k)\tilde{n}_{\rm e} = \frac{Z n_0 e^2}{m M c^2} \frac{k^2}{2} \boldsymbol{A}_{\rm l} \cdot \left[\tilde{A}(k - k_0, \omega - \omega_0) + \tilde{A}(k + k_0, \omega + \omega_0) \right] \tag{16}$$

where $D(k,\omega) = \omega^2 - c^2 k^2 - \omega_{\rm pe}^2 \left(\frac{\omega^2}{\omega^2 - \omega_{\rm c}^2} \right)$, $v_{\rm os} = \frac{e A_{\rm l}}{mc}$ and $\omega_{\rm pi} = \omega_{\rm pe} \left(\frac{Zm}{M} \right)^{\frac{1}{2}}$.

Now we use Eq. (15) to eliminate $\tilde{\boldsymbol{A}}(k-k_0, \omega-\omega_0)$ and $\tilde{\boldsymbol{A}}(k+k_0, \omega+\omega_0)$ from Eq. (16). considering lower plasma frequency ($\omega \ll \omega_0$) and neglecting the higher non-resonant terms, then gives dispersion relation

$$\omega^2 - c_{\mathrm{s}}^2 k = \frac{k^2 v_{\mathrm{os}}^2}{4} \omega_{\mathrm{pi}}^2 \left(\frac{\omega_0^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right) \times \times \left[\frac{1}{D(\omega-\omega_0, k-k_0)} + \frac{1}{D(\omega+\omega_0, k+k_0)} \right], \quad (17)$$

where the dispersion relation for incident light wave is

$$\omega_0^2 - c^2 k_0^2 - \omega_{\mathrm{pe}}^2 \left(\frac{\omega_0^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right) = 0.$$

For Brilloin back scattering, only the downshifted light wave is resonant, [1] then

$$\left(\omega^2 - c_{\mathrm{s}}^2 k\right)\left(\omega^2 - 2\omega\omega_0 + 2c^2 \boldsymbol{k}\cdot\boldsymbol{k}_0 - c^2 k^2\right) = \frac{k^2 v_{\mathrm{os}}^2}{4} \omega_{\mathrm{pi}}^2 \left(\frac{\omega_0^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right). \quad (18)$$

For founding the growth rate of Brilloin back scattering (BBS) instability (γ),we assume $\omega = kc_{\mathrm{s}} + i\gamma$ where $\gamma \ll kc_{\mathrm{s}}$, Eq. (9) becomes

$$2i\gamma kc_{\mathrm{s}}(-2i\gamma\omega_0 - 2k\omega_0 c_{\mathrm{s}} + 2kk_0 c^2 - c^2 k^2) = \frac{k^2 v_{\mathrm{os}}^2}{4} \omega_{\mathrm{pi}}^2 \left(\frac{\omega_0^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right)$$

and the we can obtain the growth rate for (BBS) as below,

$$\gamma = \frac{1}{2\sqrt{2}} \frac{k_0 v_{\mathrm{os}} \omega_{\mathrm{pi}}}{\sqrt{\omega_0 k_0 c_{\mathrm{s}}}} \left(\frac{\omega_0^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right), \quad (19)$$

$$k_0 = \frac{1}{c} \left(\omega_0^2 - \frac{\omega_0^2 \omega_p^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right)^{\frac{1}{2}}, \quad (20)$$

where k_0 is the wave number of incident light. By combining the (19) and (20) we can find the growth rate of BBS as below,

$$\gamma = a\omega_{\mathrm{pi}} \left(\frac{c}{8c_{\mathrm{s}}} \right)^{\frac{1}{2}} \left[1 - \frac{\omega_{\mathrm{pe}}^2}{\omega_0^2 - \omega_{\mathrm{c}}^2} \right]^{\frac{1}{4}} \quad (21)$$

and

$$\frac{\gamma}{\omega_{\mathrm{pi}}} = a \left(\frac{c}{8c_{\mathrm{s}}} \right)^{\frac{1}{2}} \left[1 - \frac{(\omega_{\mathrm{pe}}^2/\omega_{\mathrm{c}}^2)}{(\omega_0^2/\omega_{\mathrm{c}}^2) - 1} \right]^{\frac{1}{4}}. \quad (22)$$

Figure 1 represent the variation of BBS growth rate by variation in plasma frequency in both magnetized and unmagnetized plasma where $\omega_c/\omega_0 = 0.3$, $a = 0.1$, $c/8c_s = 10^4$.

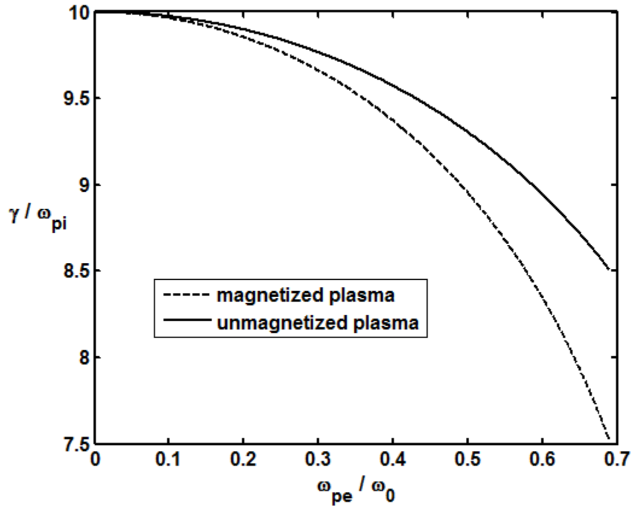

Fig. 1. Growth rate of BBS in magnetized and unmagnetized plasma

Figure 2 shows the variation of BBS growth rate by variation in electron cyclotron frequency in magnetized plasma where $\omega_p/\omega_0 = 0.3$, $c/8c_s = 10^4$, $a = 0.1$.

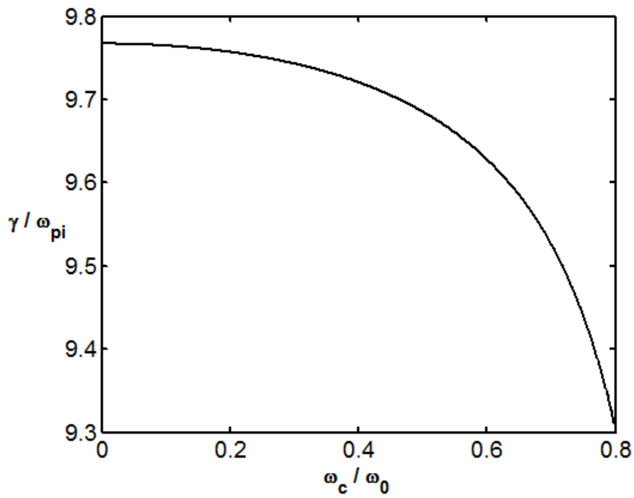

Fig. 2. Growth rate of BBS in magnetized plasma

Conclusions

About some other works associated with present work, Bawaaneh has investigated the problem of stimulated Brillouin scattering of an extraordinary light wave from a magnetized plasma[6]. He has shown that small values of static magnetic field increases the instability growth rate, while high magnetic fields reduce the instability bringing it to zero at a cut-off field. In our present work, we have considered the Brillouin scattering of an ordinary electromagnetic wave from a vertically magnetized plasma. The growth rate of Brillouin instability in the interaction of high power short laser pulse with an underdense plasma in the presence of external magnetic field ($\boldsymbol{B}_0 \perp \boldsymbol{k}_0$) is investigated. Results show a decreasing in the growth rate of Brillouin instability due to external magnetic field in comparison with unmagnetized plasma. Also growth rate is decreased when the external magnetic field is increased.

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Received May 29, 2010